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Electromagnetic absorption in an electron–hole double-layer system in a magnetic field

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Abstract. The high-frequency magnetoconductivity for a double-layer electron–hole system is calculated at finite temperature. The formalism rests on the temperature Green's function and Kubo's formula for conductivity. It is found that at frequencies higher than the cyclotron frequencies of electrons and holes, the frequency-dependent electromagnetic absorption exhibits oscillatory behaviour. The maxima in the absorption coefficient correspond to a situation where the photon energy is approximately the sum of the energies of the electron pair and the hole pair which are simultaneously excited by the radiation field.

1. Introduction

Recently, there has been considerable interest, both theoretically and experimentally, in the coupling effects of two isolated, parallel quasi-two-dimensional electron gases [1–10]. For the simplest such structure, the double quantum well (DQW), recent experiments [1] have suggested that interwell interaction can dramatically alter the single-electron levels in samples with thin tunnelling barriers. Even in a DQW with negligible tunnelling, new fractional-filling states in the extreme quantum limit have been investigated on the basis of the interlayer electron–electron interaction [2, 3]. Such a direct mechanism is also believed to play a crucial role in the interlayer momentum relaxation at low temperatures and for short interlayer separations [7–9]. Furthermore, it has been shown [11] that at low electron concentration, the interlayer Coulomb interaction is particularly strong, and hence that one must take into account the short-range exchange–correlation effect in order to remove the consistent discrepancy of several orders of magnitude in the transresistivity between the experiment and the RPA calculation. More recently, such a double-layer system was used to study the Coulomb gap in the interlayer tunnelling in a perpendicular magnetic field [12–14].

While the DC transresistivity (with or without a quantizing magnetic field) has been extensively studied theoretically, there has been very little work done on the frequency-dependent interlayer momentum and energy relaxation. Due to the development of state-of-the-art techniques such as application of free-electron lasers, investigation of electron transport and electromagnetic absorption in the high-frequency region (of the order of terahertz) has been made possible. Therefore, knowledge of the high-frequency transport is increasingly required. In this paper we will study the frequency-dependent electromagnetic absorption (or optical transresistivity) for an electron–hole double-layer system subject to a perpendicular constant magnetic field. The quantity that we will calculate is the real

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part of the optical conductivity $\mathrm{Re}[\sigma_{xx}(\omega)]$ which is a direct measure of the energy loss of photons in the double-layer system. We found that the absorption coefficient exhibits pronounced oscillations due to the discrete nature of the excitation energy of the electron pair and the hole pair (or more appropriately, the pair excitations on the electron layer and the hole layer). This discrete excitation energy causes the long-wavelength photon to be resonantly absorbed when its energy is in the vicinity of the total energy of the electron pair and the hole pair. The present electron–hole structure is an ideal system in which to study the frequency- and magnetic-field-dependent photon absorption. Owing to the difference in charge and mass between the electron and the hole, the electron–hole interaction can provide charged particles with simultaneous energy and momentum relaxations.

2. General formalism

Let us consider a double-layer system containing an electron layer and a hole layer. The separation between these two layers is taken to be a. We will ignore the interlayer tunnelling. The coupling due to Coulomb interaction will be taken into account in both the interlayer momentum relaxation and the selfconsistent screening. The system is subject to a constant magnetic field along a direction perpendicular to the layer which is taken to be the z-axis. In the Landau gauge, the single-particle wavefunction can be written as

$$\psi^s_{n,p_y}(\boldsymbol{r}) = \mathrm{e}^{\mathrm{i}p_y y}\phi^s_n(x - X) \tag{1}$$

where $\phi_n(x - X)$ is the solution of a linear harmonic system, $X = p_y l^2$ is the centre coordinate of the cyclotron motion and $l = \sqrt{1/eB}$ is the magnetic length. Here we have set $\hbar = c = k_B = 1$ for notational convenience. The spin splitting has been ignored throughout this paper [15]. We shall also use $\alpha = (n, p_y)$ as the composite quantum number. The layer index s can be either e (for the electron layer) or h (for the hole layer). The single-particle energies are simply given by the Landau levels $\epsilon^s_\alpha = (n + 1/2)\omega_{cs}$. Here $\omega_{cs} = |e_s|B/m_s$.

In the second-quantized notation, the Hamiltonian of the many-body system can be written as

$$H = H_0 + H_I \tag{2}$$

where

$$H_0 = \sum_{\alpha,s} \epsilon^s_\alpha c^\dagger_{\alpha s} c_{\alpha s}. \tag{3}$$

The interaction energy H_I is given as

$$H_I = \frac{1}{2}\sum_{\alpha,\alpha'}\sum_{\beta,\beta'}\sum_{qss'} V^{ss'}_{\alpha\alpha',\beta\beta'}(q) c^\dagger_{\alpha s} c^\dagger_{\beta s'} c_{\beta' s'} c_{\alpha' s} \tag{4}$$

where $c^\dagger_{\alpha s}$ ($c_{\alpha s}$) denotes the creation (annihilation) operator for the state of quantum number α on layer s, and these operators obey the following anti-commutation rule:

$$\left\{c_{\alpha s}(\tau), c^\dagger_{\alpha' s}(\tau')\right\}_{\tau=\tau'} = \delta_{\alpha,\alpha'}\delta_{ss'}. \tag{5}$$

In equation (4), $V^{ss'}_{\alpha\alpha',\beta\beta'}(q)$ is the interaction matrix, given as

$$V^{ss'}_{\alpha\alpha',\beta\beta'}(q) = V^{ss'}(q) F_{nn'}(Y_q) F_{m'm}(Y_q) \tag{6}$$

where $V^{ss}(q) = 2\pi e^2/\kappa q$ and $V^{ss'}(q) = -(2\pi e^2/\kappa q)\mathrm{e}^{-qd}$ if $s \neq s'$. κ is the background dielectric function. The form factor $F_{nn'}(Y)$ is given as

$$F_{nn'}(Y_q) = \sqrt{\frac{2^{n_>}n_<!}{2^{n_<}n_>!}}\left[(q_y + \mathrm{i}q_x)\frac{l}{2}\right]^{n_>-n_<} \exp\left\{\frac{\mathrm{i}l^2}{2}(q_y - 2k_y)q_x\right\}\mathrm{e}^{-Y/2}L_{n_<}^{n_>-n_<}(Y_q)\delta_{k_y-k'_y,q_y} \tag{7}$$

where $n_<$ ($n_>$) is the smaller (larger) of n and n' and $Y_q = q^2l^2/2$. $L_n^{n'}(Y_q)$ is an associated Laguerre polynomial.

We now write down the finite-temperature Green's function for the system:

$$G_{\alpha\alpha's}(\tau, \tau') = -\langle T_\tau\{c_{\alpha s}(\tau)c^\dagger_{\alpha's}(\tau')\}\rangle \tag{8}$$

where T_τ is the time ordering operator. On performing a partial Fourier transform with respect to $\tau - \tau'$, the Green's function in the energy representation is given as $G_{\alpha\alpha',s}(\zeta_l)$ with $\zeta_l = (2l+1)\pi k_B T$ (l = any integer) being the Matsubara frequency:

$$G_{\alpha\alpha's}(\zeta_l) = \frac{1}{2}\int_{-1/T}^{1/T} \mathrm{d}(\tau - \tau')\ \mathrm{e}^{\mathrm{i}\zeta_l(\tau-\tau')}G_{\alpha\alpha's}(\tau, \tau'). \tag{9}$$

For a noninteracting system, the Green's function is clearly diagonal in both the Landau level index and the centre coordinate:

$$G^0_{\alpha\alpha's}(\zeta_l) = \frac{\delta_{\alpha\alpha'}}{\mathrm{i}\zeta_l - \epsilon_{\alpha s}}. \tag{10}$$

3. Evaluation of the frequency-dependent conductivity

Within the linear response theory, the ensemble-averaged electrical current density can be written as

$$J_\mu(\boldsymbol{r}, t) = -\sum_s \frac{n_s e^2}{m_s}A_\mu(\boldsymbol{r}, t) + \mathrm{i}\int \mathrm{d}^2r' \int_0^\infty \mathrm{d}t'\ \langle[j_\mu(\boldsymbol{r}, t'),\ j_\nu(\boldsymbol{r}', 0)]\rangle A_\nu(\boldsymbol{r}', t - t'). \tag{11}$$

The nonchromatic electromagnetic radiation field is given as

$$\boldsymbol{A}(\boldsymbol{r}, t) = \boldsymbol{A}(\boldsymbol{k}, \omega)\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}-\mathrm{i}\omega t}$$
$$\boldsymbol{E}(\boldsymbol{r}, t) = \mathrm{i}\omega\boldsymbol{A}(\boldsymbol{r}, t).$$

Substituting $\boldsymbol{A}(\boldsymbol{r}, t)$ in equation (11), we obtain the wavenumber-dependent current density:

$$J_\mu(\boldsymbol{k}, t) = \left\{-\sum_s \frac{n_s e^2}{m_s}\delta_{\mu\nu} + \mathrm{i}\int_0^\infty \mathrm{d}t'\ \mathrm{e}^{\mathrm{i}\omega t'}\langle[j_\mu(\boldsymbol{k}, t'),\ j_\nu(-\boldsymbol{k}, 0)]\rangle\right\} A_\nu(\boldsymbol{k}, t - t') \tag{12}$$

which leads to the following result for the conductivity:

$$\sigma(\boldsymbol{k}, \omega) = \left\{\sum_s \frac{n_s e^2}{m_s\omega}\delta_{\mu\nu} + \frac{1}{\omega}\int_0^\infty \mathrm{d}t'\ \mathrm{e}^{\mathrm{i}\omega t'}\langle[j_\mu(\boldsymbol{k}, t'),\ j_\nu(-\boldsymbol{k}, 0)]\rangle\right\}. \tag{13}$$

The current density operator of the double-layer system is defined as

$$j_\mu(\boldsymbol{r}) = \sum_s \frac{e_s}{2\mathrm{i}m_s}\left\{\psi_s^\dagger(\boldsymbol{r})\frac{\partial\psi_s(\boldsymbol{r})}{\partial x_\mu} - \frac{\partial\psi_s^\dagger(\boldsymbol{r})}{\partial x_\mu}\psi_s(\boldsymbol{r})\right\} - \sum_s \frac{e^2}{m_s}\tilde{A}_\mu\psi_s^\dagger(\boldsymbol{r})\psi_s(\boldsymbol{r}) \tag{14}$$

where $\tilde{A}$ is the vector potential for the static magnetic field, and $\psi_s(\boldsymbol{r})$ ($\psi_s^\dagger(\boldsymbol{r})$) is the destruction (creation) field operator for the s-layer:

$$\psi_s(\boldsymbol{r}) = \sum_\alpha c_{\alpha s}\phi_n^s(x - X). \tag{15}$$

We now obtain

$$j_\mu(\boldsymbol{k}) = \sum_{\alpha\beta s} \frac{e_s}{m_s} c_{\alpha s}^\dagger c_{\beta s}\delta_{p_y', p_y + k_y} Z_{\alpha\beta s}^\mu(\boldsymbol{k}) \tag{16}$$

where

$$Z_{\alpha\beta s}^x(\boldsymbol{k}) = \int \mathrm{d}x\ \mathrm{e}^{-\mathrm{i}k_x x}\phi_n^s(-\mathrm{i}\,\partial/\partial x - k_x)\phi_{n'}^s$$

$$Z_{\alpha\beta s}^y(\boldsymbol{k}) = \int \mathrm{d}x\ \mathrm{e}^{-\mathrm{i}k_x x}\phi_n(p_y + k_y/2 - eBx)\phi_{n'}.$$

The current–current correlation function can now be written as

$$\begin{aligned}\Phi_{\mu\nu}(\boldsymbol{k}, \omega) &= \int_0^\infty \mathrm{d}t'\ \mathrm{e}^{\mathrm{i}\omega t'}\langle[j_\mu(\boldsymbol{k}, t'), j_\nu(-\boldsymbol{k}, 0)]\rangle \\ &= \sum_{\alpha\alpha'\beta\beta' ss'} \frac{e_s e_{s'}}{m_s m_{s'}} Z_{\alpha\beta s}^\mu(\boldsymbol{k}) K_{\alpha\beta s;\alpha'\beta' s'}(\omega) Z_{\alpha'\beta' s'}^\nu(\boldsymbol{k})\end{aligned} \tag{17}$$

where

$$K_{\alpha\beta s;\alpha'\beta' s'}(\omega) = \mathrm{i}\int_0^\infty \mathrm{d}t\ \mathrm{e}^{\mathrm{i}\omega t}\langle[c_{\alpha s}^\dagger(t)c_{\beta s}(t), c_{\alpha' s'}^\dagger(0)c_{\beta' s'}(0)]\rangle \qquad (\omega = \omega + \mathrm{i}0). \tag{18}$$

The conductivity components are now given as

$$\begin{aligned}\sigma_{xx}(\boldsymbol{k}, \omega) = \sum_s \frac{\mathrm{i}ne^2}{m_s\omega} + \frac{1}{\omega}\sum_{\alpha\beta s;\alpha'\beta' s'} \frac{e_s e_{s'}}{m_s m_{s'}} \\ \times \left[\frac{\mathrm{i}}{2l}[\sqrt{n+1}F_{n,n'+1}^*(Y_k) - \sqrt{n'}F_{n,n'-1}^*(Y_k)] - k_x\right] \\ \times \left[\frac{\mathrm{i}}{2l}[\sqrt{m'+1}F_{m,m'+1}(Y_k) - \sqrt{m'}F_{m,m'-1}(Y_k)] - k_x\right] \\ \times K_{\alpha\beta s;\alpha'\beta' s'}(\omega)\delta_{p_{y'}, p_y + k_y}\end{aligned} \tag{19}$$

$$\begin{aligned}\sigma_{xy}(\boldsymbol{k}, \omega) = \frac{1}{\omega}\sum_{\alpha\beta s;\alpha'\beta' s'} \frac{e_s e_{s'}}{m_s m_{s'}}\left[\frac{\mathrm{i}}{2l}[\sqrt{n+1}F_{n,n'+1}^*(Y_k) - \sqrt{n'}F_{n,n'-1}^*(Y_k)] - k_x\right] \\ \times \left[(p_y + k_y/2)F_{m,m'}(Y_k) - \frac{1}{2l}[\sqrt{m'+1}F_{m,m'+1}(Y_k)]\right. \\ \left. + \sqrt{m'}F_{m,m'-1}(Y_k)\right] K_{\alpha\beta s;\alpha'\beta' s'}(\omega)\delta_{p_{y'}, p_y + k_y}\end{aligned} \tag{20}$$

where $Y_k = (kl)^2/2$. The function $K(\omega)$ can also be written as [17, 18]

$$K_{\alpha\beta s;\alpha'\beta' s'}(\omega) = \mathrm{i}\int \frac{\mathrm{d}\omega}{\omega' - \omega}\ (1 - \mathrm{e}^{\omega'/T})\Psi_{\alpha\beta s;\alpha'\beta' s'}(\omega) \tag{21}$$

and

$$\begin{aligned}\Psi_{\alpha\beta s;\alpha'\beta' s'}(\omega) = \sum_{\gamma\gamma' s_1 s_2} \exp((\Omega - \mu N_{\gamma, s_1} - E_{\gamma, s_1})/T)\langle\gamma s_1|c_{\alpha s}^\dagger(0)c_{\beta s}(0)|\gamma' s_2\rangle \\ \times \langle\gamma' s_2|c_{\alpha s}^\dagger(0)c_{\beta s}(0)|\gamma s_1\rangle\delta(\omega - E_{\gamma', s_2} + E_{\gamma, s_1})\end{aligned} \tag{22}$$

where $\mu_s(N_s)$ is the chemical potential (particle number) of the s-layer. We now define the finite-temperature Green's function:

$$M_{\alpha\beta s;\alpha'\beta's'}(u) = \langle T_u\{c^\dagger_{\alpha s}(u)c_{\beta s}(u)c^\dagger_{\alpha s}(0)c_{\beta s}(0)\}\rangle \qquad -T^{-1} < u < T^{-1} \tag{23}$$

where

$$\langle O\rangle = \mathrm{Tr}\left\{\exp\left(\left(\Omega + \sum_s \mu_s N_s + H\right)\Big/T\right)O\right\}$$

and T_u is the time ordering operator, and

$$c^\dagger_{\alpha s}(u) = \exp\left(u\left(H - \sum_s \mu_s N_s\right)\right)c^\dagger_{\alpha s}\exp\left(-u\left(H - \sum_s \mu_s N_s\right)\right).$$

It is easy to show [17] that the Fourier transformation of $M(u)$ is directly related to the function $\Psi(\omega)$ via

$$M_{\alpha\beta s;\alpha'\beta's'}(\omega_n) = \int_0^{1/T} \mathrm{d}u\ \mathrm{e}^{\mathrm{i}\omega_n u}M_{\alpha\beta s;\alpha'\beta's'}(u) = \int \frac{\mathrm{d}\omega'}{\omega' - \omega_n}\ (1 - \mathrm{e}^{-\omega'/T})\Psi_{\alpha\beta s;\alpha'\beta's'}(\omega'). \tag{24}$$

Next we shall evaluate the finite-temperature Green's function $M(\omega_n)$ and then carry out the analytical continuation to obtain the current correlation function $K(\omega)$. The method for calculating $M(\omega_n)$ in the perturbation theory is standard and well documented [16, 17]. We shall only outline the essential steps for applying the method to the double-layer system.

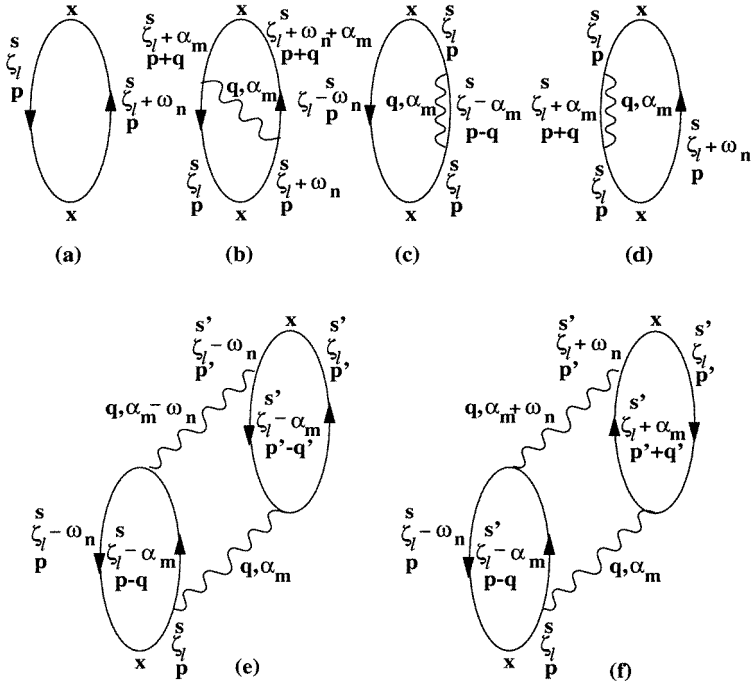

Figure 1. A class of diagrams which contribute to the high-frequency conductivity.

In the interaction picture, the Green's function can be written as

$$M_{\alpha\beta s;\alpha'\beta's'}(u) = \langle T_u\{c^\dagger_{\alpha s}(u)c_{\beta s}(u)c^\dagger_{\alpha' s'}(0)c_{\beta' s'}(0)U(0)\}\rangle_0/\langle U(0)\rangle_0. \tag{25}$$

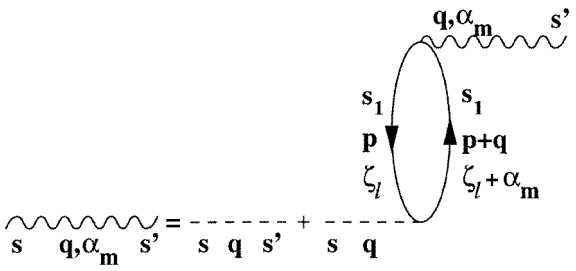

Figure 2. The effective Coulomb interaction.

Here

$$\langle O\rangle_0 = \mathrm{Tr}\left\{\exp\left(-\left(H_0 - \sum_s \mu_s N_s\right)\Big/T\right)O\right\}\Big/\mathrm{Tr}\left\{\exp\left(-\left(H_0 - \sum_s \mu_s N_s\right)\Big/T\right)\right\}$$

and

$$U(0) = \sum_0 \frac{(-1)^n}{n!}\int_0^{1/T} \mathrm{d}u_1 \ \cdots \int_0^{1/T} \mathrm{d}u_n \ T_u(H_I(u_1)\cdots H_I(u_n)). \tag{26}$$

A perturbation expansion for $M(u)$ can be obtained with the use of Wick's theorem. We then sum again all diagrams which contribute to the conductivity when the number of particles in the Bohr sphere is large. In the *high-frequency and long-wavelength limit*, only the diagrams given in figure 1 contribute to the conductivity. In figure 1, solid lines represent the electron propagators, crosses indicates external vertices [16], and wavy lines represent the fully screened Coulomb interactions as indicated in figure 2 and given by the following integral equations:

$$v^{ss'}(q,\alpha_m) = V^{ss}(q) + \sum_{s''} V^{ss''}(q)\Pi_{s''}(q,\alpha_m)v^{s''s'}(q,\alpha_m) \tag{27}$$

where

$$\Pi_s(q,\alpha_m) = T\sum_{\alpha\alpha'}|F_{\alpha\alpha'}(q)|^2\sum_l G_\alpha(\zeta_l)G_{\alpha'}(\zeta_l+\alpha_m). \tag{28}$$

Here $\alpha_m = \mathrm{i}\,2m\pi T$ is a boson Matsubara frequency with m being any integer. Equation (27) can be solved to yield the following expressions for the effective Coulomb interactions:

$$v^{ee}(q,\omega) = \frac{V^{ee}(q)(1-(V^{hh}(q)-V^{he}(q))\Pi_h(q,\omega))}{D(q,\omega)} \tag{29}$$

and

$$v^{he}(q,\omega) = \frac{V^{he}(q)}{D(q,\omega)} \tag{30}$$

and similar expressions for v^{hh} and v^{eh}. In the above equation, $D(q,\omega)$ is the determinant of the dielectric tensor, given as

$$\begin{aligned} D(q,\omega) = {} & (1-V^{ee}(q)\Pi_e(q,\omega))(1-V^{hh}(q)\Pi_h(q,\omega)) \\ & - V^{eh}(q)V^{he}(q)\Pi_e(q,\omega)\Pi_h(q,\omega). \end{aligned} \tag{31}$$

The optical conductivity $\sigma(0,\omega)$ is now given as

$$\sigma_{xx}(\omega)=\frac{\mathrm{i}ne^2}{m\omega}+\frac{1}{\omega}\sum_{\alpha\beta s;\alpha'\beta's'}\frac{e_se_{s'}}{m_sm_{s'}}\left[\frac{\mathrm{i}}{2l}[\sqrt{n+1}\delta_{n,n'+1}-\sqrt{n'}\delta_{n,n'-1}]\right]\times\left[\frac{\mathrm{i}}{2l}[\sqrt{m'+1}\delta_{m,m'+1}-\sqrt{m'}\delta_{m,m'-1}]\right]M_{\alpha\beta s;\alpha'\beta's'}(\omega_n)\delta_{p_{y'},p_y} \tag{32}$$

$$\sigma_{xy}(\omega_n)=\frac{1}{\omega_n}\sum_{\alpha\beta s;\alpha'\beta's'}\frac{e_se_{s'}}{m_sm_{s'}}\left[\frac{\mathrm{i}}{2l}[\sqrt{n+1}\delta_{n,n'+1}-\sqrt{n'}\delta_{n,n'-1}]\right]\times\left[p_y\delta_{m,m'}-\frac{1}{2l}[\sqrt{m'+1}\delta_{m,m'+1}+\sqrt{m'}\delta_{m,m'-1}]\right]M_{\alpha\beta s;\alpha'\beta's'}(\omega_n)\delta_{p_{y'},p_y}. \tag{33}$$

The absorption of electromagnetic waves by the electron–hole system is only determined by $\sigma_{xx}(\omega)$ which will be evaluated below (the Hall conductivity σ_{xy} can be calculated in a similar manner). We now calculate the conductivity according to the diagrams given in figure 1. The contribution from diagram (a) is simply

$$\sigma_{xx}^{a}=\frac{1}{\omega}\sum_s\frac{e^2n}{m_s}\frac{2\omega_{cs}^2}{\omega_{cs}^2-\omega_n^2} \tag{34}$$

and the contribution to the optical conductivity from diagrams (b)–(d) is given as

$$\sigma_{xx}^{b-d}=\frac{1}{\omega}\sum_s\frac{e_s^2}{m_s^2}\frac{1}{(\omega_{cs}^2-\omega_n^2)^2}\sum_q(q_x^2\omega_n^2+q_y^2\omega_{cs}^2)T\sum_m v^{ss}(q,\alpha_m)\times\left[\Pi_s(q,\alpha_m+\omega_n)-\Pi_s(q,\alpha_m)\right]. \tag{35}$$

With the use of equation (27), it can be rewritten in a more symmetric form:

$$\sigma_{xx}^{b-d}=-\frac{1}{\omega}\sum_{ss'}\frac{e_s^2}{m_s^2}\frac{1}{(\omega_{cs}^2-\omega_n^2)^2}\sum_q(q_x^2\omega_n^2+q_y^2\omega_{cs}^2)\times T\sum_m v^{ss'}(q,\alpha_m)v^{s's}(q,\alpha_m+\omega_n)\left[\Pi_s(q,\alpha_m+\omega_n)-\Pi_s(q,\alpha_m)\right]\times\left[\Pi_{s'}(q,\alpha_m+\omega_n)-\Pi_{s'}(q,\alpha_m)\right]. \tag{36}$$

The contribution from diagrams (e) and (f) can be written as

$$\sigma_{xx}^{e-f}=\frac{1}{\omega}\sum_{ss'}\frac{e_se_{s'}}{m_sm_{s'}}\frac{1}{(\omega_{cs}^2-\omega_n^2)(\omega_{cs'}^2-\omega_n^2)}\sum_q(q_x^2\omega_n^2+q_y^2\omega_{cs}\omega_{cs'})\times T\sum_m v^{ss'}(q,\alpha_m)v^{s's}(q,\alpha_m+\omega_n)\left[\Pi_s(q,\alpha_m+\omega_n)-\Pi_s(q,\alpha_m)\right]\times\left[\Pi_{s'}(q,\alpha_m+\omega_n)-\Pi_{s'}(q,\alpha_m)\right]. \tag{37}$$

The optical conductivity can now be written as

$$\sigma_{xx}=\sum_s\frac{\mathrm{i}ne^2}{m_s\omega}-\sum_s\frac{\mathrm{i}ne^2\omega_{cs}^2}{m_s\omega(\omega_{cs}^2-\omega_n^2)}+\frac{1}{\omega}\sum_{ss'\boldsymbol{q}}\frac{e_s}{m_s}\frac{1}{(\omega_{cs}^2-\omega_n^2)}\times\left\{\left[\frac{e_s}{m_s(\omega_{cs}^2-\omega_n^2)}-\frac{e_{s'}}{m_{s'}(\omega_{cs'}^2-\omega_n^2)}\right]q_x^2\omega_n^2+\left[\frac{e_s\omega_{cs}}{m_s(\omega_{cs}^2-\omega_n^2)}-\frac{e_{s'}\omega_{cs'}}{m_{s'}(\omega_{cs'}^2-\omega_n^2)}\right]q_y^2\omega_{cs}\right\}$$

$$\times\ T\sum_{m} v^{ss'}(q,\alpha_m)v^{s's}(q,\alpha_m+\omega_n)\left[\Pi_s(q,\alpha_m+\omega_n)-\Pi_s(q,\alpha_m)\right]$$
$$\times\ \left[\Pi_{s'}(q,\alpha_m+\omega_n)-\Pi_{s'}(q,\alpha_m)\right]. \tag{38}$$

The above result shows cancellation of all contributions which do not simultaneously have an electron pair and a hole pair in their final state. Thus, the intralayer interactions (electron–electron and hole–hole) do not contribute to a long-wavelength photon absorption, except in the selfconsistent screening.

We now first perform the summation over m, then the analytical continuation to obtain

$$\sigma_{xx}(\omega)=\sum_s\frac{\mathrm{i}ne^2}{m_s\omega}-\sum_s\frac{\mathrm{i}ne^2\omega_{cs}^2}{m_s\omega(\omega_{cs}^2-\omega^2)}+\frac{1}{\omega}\sum_{ss'\boldsymbol{q}}\frac{e_s}{m_s}\frac{1}{(\omega_{cs}^2-\omega^2)}|V^{ss'}(q)|^2$$
$$\times\ \left\{\left[\frac{e_s}{m_s(\omega_{cs}^2-\omega^2)}-\frac{e_{s'}}{m_{s'}(\omega_{cs'}^2-\omega^2)}\right]q_x^2\omega^2\right.$$
$$\left.+\ \left[\frac{e_s\omega_{cs}}{m_s(\omega_{cs}^2-\omega^2)}-\frac{e_{s'}\omega_{cs'}}{m_{s'}(\omega_{cs'}^2-\omega^2)}\right]q_y^2\omega_{cs}\right\}\frac{\mathrm{P}}{2\pi\mathrm{i}}\int\mathrm{d}x\ \coth\left(\frac{x}{2T}\right)$$
$$\times\left\{\frac{\left[\Pi_s(q,x^+\!+\omega)-\Pi_s(q,x^+)\right]\left[\Pi_{s'}(q,x^+\!+\omega)-\Pi_{s'}(q,x^+)\right]}{D(q,x^+)D(q,x^+\!+\omega)}\right.$$
$$\left.\times\ \frac{\left[\Pi_s(q,x^+\!+\omega)-\Pi_s(q,x^-)\right]\left[\Pi_{s'}(q,x^+\!+\omega)-\Pi_{s'}(q,x^-)\right]}{D(q,x^-)D(q,x^+\!+\omega)}\right\}. \tag{39}$$

Here, P stands for the principal value and $x^{\pm}=x\pm\mathrm{i}\eta$ ($\eta\to 0$). We now write the conductivity in terms of electrons and holes:

$$\sigma_{xx}(\omega)=\sigma_{xx}^0(\omega)+\sigma_{xx}^1(\omega) \tag{40}$$

where

$$\sigma_{xx}^0(\omega)=\mathrm{i}ne^2\omega\left[\frac{1}{m_e(\omega^2-\omega_{ce}^2)}+\frac{1}{m_h(\omega^2-\omega_{ch}^2)}\right] \tag{41}$$

and

$$\sigma_{xx}^1(\omega)=\frac{1}{\omega}\sum_{\boldsymbol{q}}|V^{eh}(q)|^2\left\{\left[\frac{e}{m_e(\omega^2-\omega_{ce}^2)}+\frac{e}{m_h(\omega^2-\omega_{ch}^2)}\right]^2q_x^2\omega^2\right.$$
$$\left.+\ \left[\frac{e\omega_{ce}}{m_e(\omega^2-\omega_{ce}^2)}+\frac{e\omega_{ch}}{m_h(\omega^2-\omega_{ch}^2)}\right]^2q_y^2\right\}\frac{\mathrm{P}}{2\pi\mathrm{i}}\int\mathrm{d}x\ \coth\left(\frac{x}{2T}\right)$$
$$\times\left\{\frac{\left[\Pi_e(q,x^+\!+\omega)-\Pi_e(q,x^+)\right]\left[\Pi_h(q,x^+\!+\omega)-\Pi_h(q,x^+)\right]}{D(q,x^+)D(q,x^+\!+\omega)}\right.$$
$$\left.\times\ \frac{\left[\Pi_e(q,x^+\!+\omega)-\Pi_e(q,x^-)\right]\left[\Pi_h(q,x^+\!+\omega)-\Pi_h(q,x^-)\right]}{D(q,x^-)D(q,x^+\!+\omega)}\right\}. \tag{42}$$

Here $\omega_{ce}=eB/m_e$ and $\omega_{ch}=eB/m_h$. It can be seen that $\sigma_{xx}^0(\omega)$ is pure imaginary and $\sigma_{xx}^1(\omega)$ is complex.

4. Electromagnetic absorption

In the following, we are only interested in the absorptive part of the conductivity, $\mathrm{Re}[\sigma_{xx}(\omega)] = \mathrm{Re}[\sigma^1_{xx}(\omega)]$, which is given as

$$\mathrm{Re}[\sigma_{xx}(\omega)] = \frac{1}{\omega}\sum_{q}|V^{eh}(q)|^2\left\{\left[\frac{e}{m_e(\omega^2-\omega_{ce}^2)}+\frac{e}{m_h(\omega^2-\omega_{ch}^2)}\right]^2 q_x^2\omega^2 + \left[\frac{e\omega_{ce}}{m_e(\omega^2-\omega_{ce}^2)}+\frac{e\omega_{ch}}{m_h(\omega^2-\omega_{ch}^2)}\right]^2 q_y^2\right\} \times \frac{\mathrm{P}}{2}\int \mathrm{d}x\ \left[\coth\left(\frac{x}{2T}\right)-\coth\left(\frac{x+\omega}{2T}\right)\right]F(x,x+\omega) \tag{43}$$

where

$$F(x,x+\omega) = 2(F_1+F_2+F_3)$$

with

$$F_1 = \left[\Pi_h^I(q,x+\omega)\Pi_e^I(q,x)+\Pi_h^I(q,x)\Pi_e^I(q,x+\omega)\right]D^{-1}(x+\omega)^R D^{-1}(x)^R$$

and

$$F_2 = \left[\Pi_h^I(q,x+\omega)\Pi_e^I(q,x+\omega)+\Pi_h^I(q,x)\Pi_e^I(q,x)\right]D^{-1}(x+\omega)^I D^{-1}(x)^I$$

and

$$F_3 = -\left[\Pi_h^R(q,x+\omega)-\Pi_h^R(q,x)\right]\left[\Pi_e^R(q,x+\omega)-\Pi_e^R(q,x)\right]D^{-1}(x+\omega)^I D^{-1}(x)^I + \left[\Pi_h^R(q,x+\omega)-\Pi_h^R(q,x)\right]\left[D^{-1}(x+\omega)^R D^{-1}(x)^I\Pi_e^I(x+\omega) - D^{-1}(x+\omega)^I D^{-1}(x)^R\Pi_e^I(x)\right]+\left[\Pi_e^R(q,x+\omega)-\Pi_e^R(q,x)\right] \times \left[D^{-1}(x+\omega)^R D^{-1}(x)^I\Pi_h^I(x+\omega)-D^{-1}(x+\omega)^I D^{-1}(x)^R\Pi_h^I(x)\right]$$

where A^R (A^I) refers to the real (imaginary) part of A, and $D^{-1}(x)^R$ should be understood as $(1/D(x))^R$. Equation (43) is the central result of this work. It can be evaluated analytically (in limiting cases only) or numerically for any value of the magnetic field, layer separation, and electron–hole mass ratio, and at any temperature. To avoid the nonphysical behaviour arising from the singular nature of the density of states, it is essential to include from the outset the level broadening due to disorder. Here we assume that the level broadening is due to the short-range scatterers which can be treated within the selfconsistent Born approximation (SCBA) [18]. In our numerical calculation, we have used the following parameters for a typical GaAs–AlGaAs-based double-layer system: $m_e = 0.067m_0$, $n_e = n_h = 3.3\times10^{11}$ cm^{-2}, $\kappa = 12.9$, $k_F a = 1.0$. The scattering rate appearing in the SCBA [18] is taken to be $\Gamma_0 = 0.07\sqrt{B}$ meV which corresponds to the electronic mobility of $\mu = 10^6$ cm^2 V^{-1} s^{-1}. We have also assumed this Γ_0 is same for both the electron layer and the hole layer. This assumption is probably not true for a real system, but is not essential to our work. The result (43) can be applied to any system when real parameters are given.

5. Results and discussion

In figure 3 we present the numerical result for the normalized high-frequency absorption coefficient $\mathrm{Re}[\sigma_{xx}(\omega)]/\sigma^0_{xx}(\omega)$. The absorption coefficient decreases with the radiation frequency as expected in a quantum electronic system. However, this overall decrease

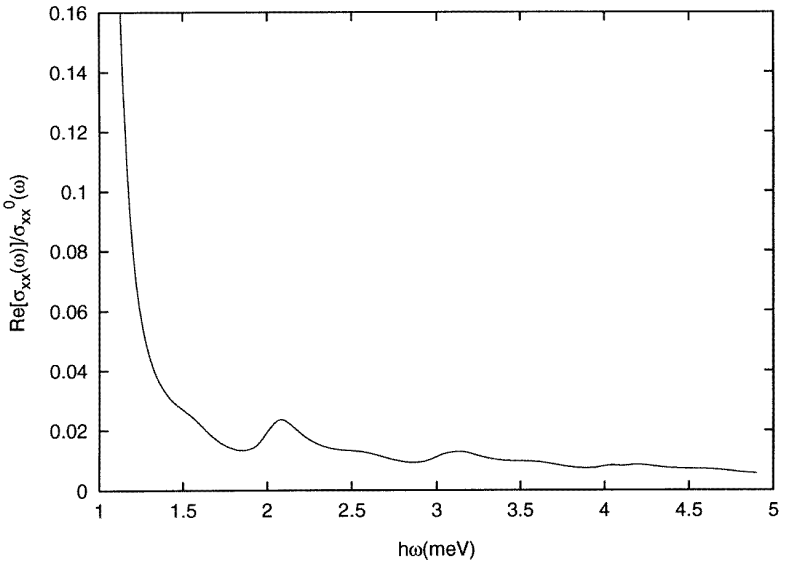

Figure 3. The real part of the optical conductivity in units of $\sigma^0_{xx}(\omega)$ as a function of the photon frequency. Here $\omega_{ce} = 1$ meV, $m_h/m_e = 2.0$, $\Gamma_0 = 0.07\sqrt{B}$ meV, $T = 2.2$ K.

with frequency in the present system is not monotonic. The absorption exhibits quantum oscillation while it decreases with the radiation frequency. The absorption has a maximum when the photon energy is approximately the sum of the excitation energy for an electron pair and a hole pair. This behaviour is depicted in figure 3 where $\omega_{ce} = 1$ meV and the mass ratio is 2, in which case the Landau level separation for the hole layer is half of that of the electron layer. Our findings are as follows.

(i) At $\omega \approx 1.5$ meV $= \omega_{ce} + \omega_{ch}$, the photon can be strongly absorbed due to the simultaneous excitation of an electron pair (of energy ω_{ce}) and a hole pair (of energy ω_{ch}).

(ii) At $\omega \approx 2$ meV, an electron pair (of energy ω_{ce}) and a hole pair (of energy $2\omega_{ch}$) can be excited and strong absorption occurs.

(iii) The maximum absorption actually occurs at energies slightly higher than the bare excitation energies of electrons and holes (e.g. the actual maxima are at 2.1, 2.55, 3.15, 3.58, 4.15 meV etc). This shift is mainly due to the frequency- and magnetic-field-dependent screening and the mixing of various contributions from F_2 and F_3. The effective-mass shift due to the self-energy correction has a rather small effect on the positions of the absorption maxima. If one applies the static screening approximation, the absorption maxima will occur at energies very close to the sum of bare excitation energies of electrons and holes.

(iv) The situation at higher frequency is slightly more complicated, as several different processes can all contribute. For example, at $\omega \approx 3$ meV (or 3.15 meV to be more precise), an electron pair of energy ω_{ce} together with a hole pair of energy $4\omega_{ch}$ can be excited. The excitation of an electron pair of energy $2\omega_{ce}$ with a hole pair of energy $2\omega_{ch}$ will also contribute to the resonant absorption at around this frequency.

In general, the positions of the maximum absorption (which are determined by $\Pi^I(\omega)$) at a given frequency arising from different processes are not exactly the same, due the same effects as are described in (iii). If this separation is large enough, a split in the absorption maximum can be observed (see figure 3 at ω around 4 meV).

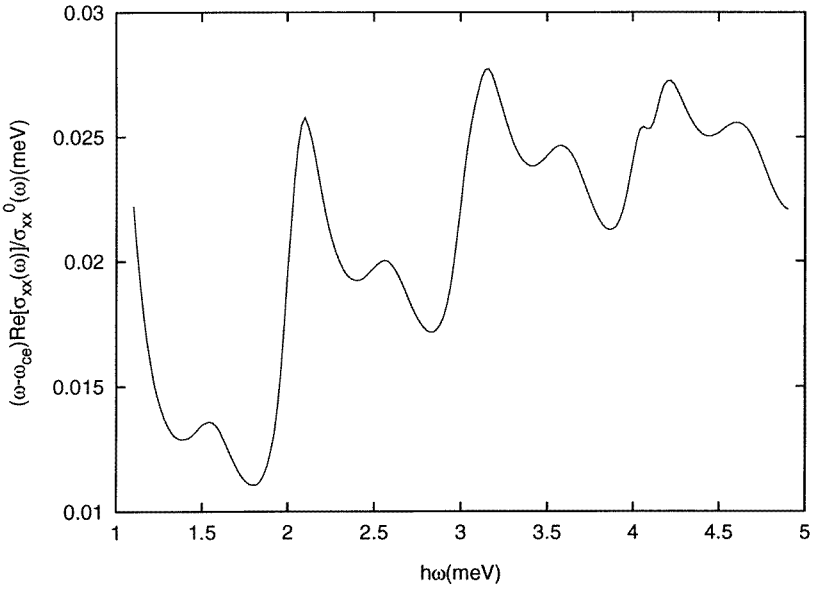

Figure 4. A plot of $(\omega - \omega_{ce})\sigma^{R}_{xx}(\omega)/\sigma^{0}_{xx}(\omega)$ as a function of the photon energy for the same parameters as were used in figure 2.

To further demonstrate these quantum oscillations, we multiply the absorption by $\omega - \omega_{ce}$ and plot the result in figure 4. This procedure effectively removes the envelope function which decreases rapidly with the frequency, and brings out clearly the oscillatory behaviour of the electromagnetic absorption.

It should be pointed out that the nonphysical behaviour of the electromagnetic absorption near ω_{ce} and ω_{ch} is an artifact of the present formalism which is essentially valid in the high-frequency regime. To extend this result to near the cyclotron energy, one needs to sum again all the ladder diagrams which contribute to the conductivity. The present result represents the leading-order expansion in the Drude-type magnetoconductivity in terms of the parameters $1/(\omega - \omega_{ce})\tau$ and $1/(\omega - \omega_{ch})\tau$. There is a recent paper [19] which deals with such contributions due to ladder diagrams. These ladder diagrams play a crucial role in the conductivity at low frequencies and especially in the DC limit. However, in the high-frequency limit, the contribution from these ladder diagrams becomes negligible [17, 18]. As has mentioned above, each additional ladder contributes a factor of $1/(\omega \pm \omega_{ce(h)})\tau$ or $\mathrm{Re}[\sigma^{1}]/\sigma^{0}$. In the frequency regime of interest, say $\omega > 1.3\omega_{ce}$, this quantity is indeed a small quantity, of the order of 3×10^{-2} (see figure 3). Therefore the lowest-order ladder diagram contributes a term of the order of 10^{-3}, a quantity that can be safely ignored. Of course, this high-frequency approximation will result in the above-mentioned nonphysical behaviour near ω_{ce}.

The result (41) exhibits full symmetry between the electrons and holes. If the hole mass can be treated as infinitely heavy, we should recover the results for an electron–impurity system. However, this limit must be approached with care. The charge-carrying holes, besides providing momentum relaxation, contribute to the electric current in two ways: (i) they carry current themselves, which becomes negligible at large hole mass, and (ii) they participate in the selfconsistent screening, which for large hole mass manifests itself in the retaining of the contribution to the static screening which influences the conductivity at low

and intermediate frequencies. As the hole mass becomes much heavier than the electron mass, we have

$$\Pi_h(q, x+\omega) \pm \Pi_h(q, x) \to \pm \mathrm{i}n\delta(x) \tag{44}$$

and we must also let Π_h be equal to zero everywhere in the dielectric function to obtain

$$\mathrm{Re}[\sigma(\omega)] = \frac{1}{2\pi m_e^2(\omega^2 - \omega_{ce}^2)^2} \sum_{\boldsymbol{q}} |V^{eh}(q)|^2 (q_x^2\omega^2 + q_y^2\omega_{ce}^2)\, \mathrm{Im}\left[\frac{1}{\epsilon(q,\omega)}\right] \tag{45}$$

where $\epsilon(q,\omega) = 1 - (2\pi e^2/\kappa q)\Pi_e(q,\omega)$. This is the familiar form of the high-frequency magnetoconductivity for an electron–impurity system.

In conclusion, we have calculated the electromagnetic absorption for an electron–hole double-layer system under a constant magnetic field. An oscillatory frequency-dependent absorption is obtained which has its origin in the resonant excitation of an electron pair and a hole pair. While most experiments were carried out in the DC limit, it is our hope that this work will stimulate experimental measurements on this interesting system in the high-frequency regime. With the latest development of state-of-the-art techniques such as the application of free-electron lasers, it is now possible to perform such a frequency-tunable measurement in the frequency regime around and above the terahertz level.

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In SCBA, the level broadening is given as $\gamma(\zeta) = \Gamma_0^2 \sum_\alpha \mathrm{Im}\, G_\alpha(\zeta)$ and $G_\alpha(\zeta) = [\zeta - \epsilon_\alpha - \Sigma(\zeta)]^{-1}$.
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